The Butterfly Did It: The Aberrant Vote for Buchanan in Palm Beach County, Florida**

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Abstract

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We show that the butterfly ballot used in Palm Beach County (PBC), Florida, in the 2000 presidential election caused more than 2,200 Democratic voters to vote by mistake for Reform candidate Pat Buchanan—a number larger than George W. Bush's certified margin of victory in Florida. We use multiple methods and several kinds of data to rule out alternative explanations for the votes Buchanan received in PBC. Among 3,049 counties across the United States where Buchanan was on the ballot, PBC has the most anomalous excess of votes for Buchanan. In PBC Buchanan's proportion of the vote on election-day ballots is four times larger than his proportion on absentee (non-butterfly) ballots, but Buchanan's proportion does not differ significantly between election-day and absentee ballots in any other Florida county. Unlike other Reform candidates in PBC, Buchanan tended to receive election-day votes in Democratic precincts and from individuals who voted for the Democratic U.S. Senate candidate. Robust estimation of overdispersed binomial regression models underpins much of the analysis.

Beginning on election day November 7, 2000, Palm Beach County, Florida, attracted national and eventually international attention because thousands of voters in the county complained that they had difficulty understanding the now infamous *butterfly ballot* and consequently cast invalid or erroneous presidential votes. Lawyers working for the Democratic party reportedly collected 10,000 affidavits sworn by voters with complaints about some aspect of their election day experiences in Palm Beach County.[1] Shortly after election day, eleven groups of Palm Beach County voters filed independent lawsuits seeking relief, claiming they and others had made mistakes in their votes for president because of the confusing format of the ballot.[2] Many of the complaining voters stated that they had intended to vote for Democratic candidate Al Gore but instead had voted by mistake for Reform candidate Pat Buchanan. The number of votes involved was more than enough to have tipped the presidential vote in Florida from Republican candidate George W. Bush to Gore, thus giving him Florida's 25 electoral votes and the presidency.[3]

Palm Beach County is a heavily Democratic, politically liberal county that conventional wisdom says should provide few Buchanan votes.[4] Two days after the election Bay Buchanan, Pat Buchanan's sister and campaign manager, said "she was startled to hear Bush strategist Karl Rove argue Thursday that Buchanan has strong support in a county where his campaign never bought an ad and never paid a visit."[5] Yet in the 2000 presidential election the county supplied 19.6% of Buchanan's votes in Florida. In contrast, only 5.4% of Buchanan's Florida votes came from Palm Beach County in the 1996 Republican presidential primary, which did not use a butterfly ballot. In the 1996 presidential election the county gave less support to Reform than did Florida as a whole, but in 2000 this pattern was dramatically reversed.[6]

Among scholars a consensus quickly formed that the vote for Buchanan in Palm Beach County was anomalously large. According to the initial, uncertified count of Palm Beach County ballots, Buchanan received 3,407 votes. Within four days of the election, more than a dozen scholars had posted to the Internet analyses of the electoral outcome in Palm Beach County.[7] The technical approaches used in the early analytical work varied greatly, but most scholars agreed that 3,407 was many times greater than the number of votes to be expected for Buchanan under normal circumstances. Given various measurements of county political configurations or demographic composition, a typical estimate was that Buchanan received about 2,800 more votes than were to be expected based on voting patterns in other Florida counties. In an especially careful study, Bruce Hansen reported 2,058 as a conservative minimum for the number of votes that were counted for Buchanan but that were irregular.[8]

Our own county-level model for Buchanan's Florida vote gives an estimate of 438 for the number of votes to be expected for Buchanan based on previous election results and demographic factors (see Table 5). That estimate is based on the final, certified election returns, in which Buchanan received 3,411 votes in Palm Beach County. In the final, certified returns Bush had a margin over Gore of only 537 votes in Florida.[9] The number of apparently accidental votes for Buchanan was greater than Bush's official margin of victory in the state. If those votes had gone to Gore, Gore would have won Florida and become President. If the butterfly ballot caused those voters mistakenly to choose Buchanan, then the butterfly ballot was pivotal in the election.

The butterfly ballot, shown in Figure 1, was an innovation of Theresa LePore, Supervisor of Elections for Palm Beach County, Florida.[10] The distinctive format was used only in Palm Beach County and only for the election-day ballots for the presidential race. It is a "butterfly" because there are two columns of names of candidates (the "wings of the butterfly"), all for the same office, that sandwich a single column of punch holes between the names. These punch holes are alternately for the left-hand and right-hand side candidates. Thus, the first valid punch hole (#3) is for Bush, the first candidate on the left-hand side of the ballot. The second valid punch hole (#4) is for Buchanan, the first candidate on the right-hand side of the ballot. But from the perspective of the left-hand side of the ballot, the second candidate listed is Gore. An individual who scanned down the left-hand column of the ballot and only later considered the right-hand column could mistakenly conclude that the first two punch holes corresponded, respectively, to Bush and Gore. Even having made such an incorrect reading of the ballot, a Bush voter would be likely to punch the hole for Bush correctly, but a Gore voter might mistakenly punch the hole for Buchanan.[11]

*** Figure 1 about here ***

Did the butterfly ballot cost Al Gore the presidency? The lawsuits filed by citizens of Palm Beach County were thrown out because the Supreme Court of Florida ruled that the butterfly ballot was not illegal.[12] But the Court's ruling did not depend upon and implied nothing about the impact the ballot actually had on the behavior of voters in Palm Beach County. Our goal in this paper is to confirm whether the butterfly ballot caused several thousand Gore supporters to vote mistakenly for Buchanan.

1 Overview

To test whether Democratic voters mistakenly voted for Buchanan, we use multiple methods and diverse data sources to rule out alternative explanations for Buchanan's vote in Palm Beach County. Our first step is to show that the outcome in Palm Beach County was irregular. To do this, we compare actual numbers of votes for Buchanan to expected numbers that we estimate from previous election results and demographic data. In Section 2 we calculate the discrepancy between actual and expected votes in 2000 for counties throughout the U.S. Our purpose is to identify counties where there are large discrepancies to see whether the apparently large vote for Patrick Buchanan in Palm Beach County is exceptional or ordinary.

Because counties differ dramatically in size (some have only a few hundred residents while others have millions), it would be statistically misleading to compare the differences between actual and expected votes directly across counties. A few Buchanan votes in a county with a few hundred voters can lead to large percentage discrepancies that should not be taken to be significant. We adjust for this and other sources of heteroscedasticity by using studentized residuals (Carroll and Ruppert, 1988, 31–34) to measure the discrepancies between actual and expected votes. The studentized residuals are comparable across counties.

The most important methodological innovation in our analysis is that we develop new methods for robust estimation of overdispersed binomial regression models.[13] In the Appendix we describe the details of our statistical technique. We use a *least quartile difference* (LQD) estimator (Rousseeuw and Croux, 1993) to estimate the dispersion and then, given the LQD dispersion estimate, we use a *hyperbolic tangent* (tanh) estimator (Hampel, Rousseeuw, and Ronchetti, 1981) to estimate the coefficient parameters. As discussed in the Appendix, our robust methods are useful for several reasons. With such methods we can be reasonably sure that deviations from the expected values are the result of idiosyncratic factors such as poor ballot design, defective voting machines or extraordinary local support for Buchanan. Also, the robust estimators produce reliable measures of discrepancy even when the regression model gives only a rough approximation for the processes that produced the vote counts, and even when some of the data used to compute the expected values are distorted.

We use our robust methods to calculate the discrepancy between actual and expected Buchanan votes for each county. If a county's discrepancy is more than four standard deviations away from zero then we declare the outcome there to be irregular: the county is an *outlier*.[14] That is, the statistical model that provides the best prediction of the current election results for the counties in the rest of the state does not do a good job

of predicting the results for the outlier county, suggesting that the results there are produced in a different way than those of the other counties.

To explain *why* a county is an outlier requires investigation using additional information. There may be many plausible explanations for the observed anomaly.[15] Our analysis of the impact of the butterfly ballot uses a strategy of triangulation to eliminate plausible alternative explanations. We show that different models using different types of data all point to the same conclusion regarding the Buchanan vote in Palm Beach County: the butterfly ballot did cause at least 2,000 voters to vote mistakenly for Buchanan, and the vast majority of these votes were from Democrats.

In Section 2 we show that the vote for Buchanan in Palm Beach County truly was unusual. If anomalies as large as the one observed in Palm Beach County are relatively frequent, then maybe it is unreasonable to credit the butterfly ballot for the unexpected outcome in Palm Beach County. We find that anomalies comparable to the one in Palm Beach County do not occur elsewhere for the 2000 election. Out of 3,049 counties across the United States—all the counties for which estimation is possible by our methods—Palm Beach County has the largest discrepancy between the actual 2000 vote for Buchanan and the vote one would expect based on each county's electoral history and demographic profile. Our robust estimation methods ensure against various threats to the statistical validity of that finding.

But perhaps the Reform party vote in Palm Beach County has been anomalous in the past. In Section 2 we also show that the result for the Reform candidate in Palm Beach County in the previous presidential election—where a butterfly ballot format was not used—was not anomalous when compared to the vote for Reform in the other 66 counties of Florida.

Nevertheless it is possible that the large Buchanan vote in Palm Beach County in 2000 is a one-time anomaly because of some idiosyncratic factor other than the butterfly ballot. If so, we would expect that the anomaly would appear in both the election-day results that used the butterfly ballot and the absentee results that did not use the butterfly ballot. In fact, because we expect absentee voters to be politically more conservative than election-day voters, we should find that the absentee results have an even larger percentage vote for Buchanan. In Section 3 we show that in Palm Beach County the reverse is true. There is a much larger percentage of the vote for Buchanan among election-day voters than among absentee voters, and the difference between the support for Buchanan among election-day voters and the support for Buchanan among absentee voters is extraordinarily large compared to the same difference in all other Florida counties.

We conclude that the anomalous results in Palm Beach County occurred only among election-day voters who used the butterfly ballot. In effect, the use of two different ballot forms in Palm Beach County presents us with a natural experiment that allows us to conclude that the unexpectedly large Buchanan vote in Palm Beach County is almost certainly the result of the butterfly ballot.

But were these Buchanan voters people who meant to vote for Al Gore? In general we expect that party loyalty and ideological positions make it likely that those voting Democratic for one office in an election will vote for Democrats in another office. When Democrats do defect, we expect that they will choose someone who is ideologically close to them, and not someone who is ideologically distant. These commonplace observations make it unlikely that those voting for the Democratic candidate in the 2000 election for U.S. Senator from Florida would vote for Reform candidates for other offices. Therefore, we would not expect to find much Buchanan support in those precincts that voted more heavily for the Democratic U.S. Senate candidate. We would expect the share of votes for Buchanan to decrease as the share of votes for the Democratic U.S. Senate candidate increases. Such expectations are wrong for Palm Beach County. In Section 4 we use precinct-level data from Palm Beach County to show that precincts that voted more heavily for the Democratic U.S. Senate candidate also voted more heavily for Buchanan for president.

The abnormality of this pattern is further underscored by the fact that precincts that voted more heavily for the Democratic U.S. Senate candidate did not tend to vote heavily for other Reform party candidates. In Section 4 we also analyze the precinct-level votes for two Reform candidates who were running, respectively, for the U.S. House and for the Florida State Senate. Unlike the case of Buchanan, the votes for these two Reform candidates decreased as votes for the Democratic U.S. Senate candidate increased. It does not appear that Democratic voters in Palm Beach County were usually attracted to Reform party candidates. Consequently, the association between Democratic votes for Senate and the vote for Buchanan is surprising. The most reasonable explanation is that the butterfly ballot caused many Democratic voters to choose Buchanan by mistake.

The vote for Buchanan in Palm Beach County is so small, however, that a few pockets of concentrated Buchanan support might explain these results. Perhaps a few highly Democratic precincts had concentrations of Buchanan supporters leading to the results described above. In such a situation, a few anomalous precincts may bias our inferences about the county as a whole. Since the butterfly ballot hypothesis concerns a county-level administrative phenomena (i.e., ballot design), a result supported only in a few anomalous

precincts would not be consistent with it. Fortunately, our robust estimation methods allow us to avoid such erroneous inferences. These methods, which we use in our Section 4 analysis, produce estimates that require much more than isolated instances of a strong relationship between Democratic precincts and Buchanan supporters. They require that the pattern must prevail across most of the precincts in the county.

In Section 5 we present an analysis of individual ballots that reconfirms the patterns found in Sections 3 and 4. Democratic voters (as measured by votes in the U.S. Senate election) who voted on election day were much more likely to vote for Buchanan than were absentee Democratic voters. These findings, as well as other patterns in the ballot-level data, support the conclusions we reached through the analysis of aggregate data: in every case, it is the butterfly ballot that best explains the unusual outcomes for Buchanan in Palm Beach County.

2 Buchanan's Vote in Counties across the United States

Our overdispersed binomial regression model predicts votes for Buchanan, given the total number of votes cast for all presidential candidates, in each county in 2000.[16] Two kinds of information, the results of previous elections and the demographic characteristics of the county, are available and highly relevant for making such predictions. The previous election outcome is a proxy not only for the array of interests and party sentiments in each county but also for the strength of local party mobilization. We use two variables to represent the preceding election result in each county: the proportion of votes officially received by the Republican candidate in the 1996 presidential election; and the proportion of votes officially received by the Reform party candidate in the 1996 presidential election. Because those two variables may not be adequate to capture the electoral environment in each county in 2000, we supplement them with a set of nine demographic variables. The demographic variables include the 2000 Census of Population and Housing proportions of county population in each of four Census Bureau race categories (White, Black, Asian and Pacific Islander, and American Indian or Alaska Native), 2000 proportion Hispanic, 2000 population density (i.e., 2000 population/1990 square miles) and 2000 population.[17] We also include the 1990 proportion of population with college degree, and 1989 median household money income. We do not use the number of voters registered as Reform party members because voter registration is nonpartisan in many states.

We cannot use all eleven variables (plus a constant) in our models at once, because with that many variables a state would need to have about 40 to 60 counties to produce reliable estimates. But 21 states

(including D.C.) have fewer than 50 counties. We take the approach of including the vote proportions variables in all of the models while using principal components of the demographic data. To maximize the efficiency of the information gained from the demographic variables, we compute principal components of the set of residuals obtained by regressing each demographic variable on the previous election proportion variables and the constant.[18] The regressions on the election variables and the computation of principal components are done separately for each state.

We compare Palm Beach County to all of the 3,049 counties in the U.S. for which we could robustly estimate the overdispersed binomial model described by Appendix equations (1) through (4) and hence compute studentized residuals.[19] The dependent variable is the count y_i of votes for Buchanan in each county i in the 2000 election.[20] We estimate the model *separately* for each state in the United States. Tests not presented here show that the coefficient and dispersion parameters cannot be pooled across states. But the studentized residuals are comparable across states.

Palm Beach County has the largest residual[21] among the 3,049 counties included in our analysis. This result can be seen in Figure 2, which presents boxplots that display the distribution of the residuals for each state.[22] Not only does the residual for Palm Beach County have the largest positive value but it is the largest in absolute magnitude. In Palm Beach County Buchanan received vastly more votes than were to be expected based on the county's electoral history and demographic profile.

*** Figure 2 about here ***

Table 5 in the Appendix lists the residuals, expected Buchanan vote proportion, actual Buchanan vote proportion and number of valid presidential ballots for all counties for which the residual is greater than 4.0 or less than -4.0. In all, the table lists 68 positive outliers and eight negative outliers. The difference between the number of positive and the number of negative outliers reflects the overall positive skew of the residuals that is visible for most states in Figure 2.[23] Using the threshold of magnitude 4.0 to classify a county as an outlier, outliers occur in 31 of the 48 states covered by our analysis.

The outlier status of some counties can readily be explained. The most easily explained outlier is Jasper County, South Carolina, which has the second largest residual in our analysis. Jasper County did not receive much national media attention, because the outcome in Jasper County was immaterial to the outcome of the 2000 presidential contest. Bush defeated Gore in South Carolina by 220,376 votes but only 6,469 presidential ballots were cast in Jasper County. Nonetheless there were serious problems with a voting

machine in Jasper County's Tillman precinct. In that precinct Gore and Bush each received one vote while Buchanan received 239 and Nader received 111.[24] The problems in the precinct affected vote totals for other offices. Indeed, "the State Board of Canvassers unanimously said [...] that problems in the county council election were so numerous that a new election should be held."[25]

Palm Beach County is not geographically contiguous to any other outlier county, but many of the counties listed in Table 5 are contiguous to another outlier.[26] Table 1 displays the sets of counties from Table 5 that are geographically contiguous.[27] Sixteen of the twenty-five largest positive outliers belong to such a geographic cluster. Two clusters includes counties from two states (West Virginia and Ohio, and Kansas and Missouri) and another includes counties from three states (South Dakota, Iowa and Nebraska). Because they span more than one state, it is highly unlikely that those exceptional bursts of support for Buchanan reflect problems of ballot format or electoral administration. Most likely the reason is special success in mobilizing voters for Buchanan in those areas.

*** Table 1 about here ***

There are also a few other outliers for which we have some information that may be relevant. For example, the outlier status of St. Louis City and St. Louis County, Missouri, may be related to a history of balloting problems in St. Louis as well as to alleged racial discrimination there in the 2000 election.[28] Possibly relevant to the positive outliers in Colorado (Arapahoe, El Paso, Jefferson and Adams) may be the profound rift in the Reform party in that state—John Hagelin, not Buchanan, was on the Colorado ballot as the Reform party candidate. Buchanan appeared as the Colorado Freedom party nominee.[29] The negative residual for Fulton County, Georgia may reflect differences in the political mobilization of blacks in Atlanta, which is located partly in Fulton, compared to black mobilization in the rest of the state.[30] De Kalb County, which also contains part of Atlanta, has a residual of $\tilde{r}_i = -3.95$.

We do not emphasize such counties here, in part because much of the information about them seems only indirectly connected to the vote for Buchanan and partly because the residuals for some of these counties are not stable over variations in the model specifications. Such instability is in contrast to the stability we see in the size and relative positions of the residuals for counties such as Palm Beach FL or Jasper SC, for which voting irregularities were reported, or Hancock WV and Pottawatomie KS which are in contiguous clusters of unusual Buchanan votes. Palm Beach County has the largest residual and Jasper County the second largest residual whether we use one or two principal components. With three principal components, Palm Beach

County has the second largest residual ($\tilde{r}_i = 35.5$) and Jasper County has the third largest ($\tilde{r}_i = 20.9$). With no principal components (i.e., only past vote proportions) Jasper County has the largest residual ($\tilde{r}_i = 25.6$) and Palm Beach County has the second largest ($\tilde{r}_i = 21.5$).

No other county in Florida comes close to Palm Beach County in terms of having an excess of votes for Buchanan in the 2000 election. The only other outlier in Florida is Pinellas County (see Table 5). The parameter estimates for Florida give a point estimate of 438 for the number of votes expected for Buchanan in Palm Beach County—implying an excess of 2,973 accidental votes in the official tally of 3,411 votes for Buchanan.

As we have tried to exemplify in our discussion of some of the outlier counties, an explanation of why a county is an outlier requires further investigation using additional information. In some counties we found clear auxiliary evidence of problems with ballots, voting machines or election administration. In still others there are strong indications that the reason Buchanan received an exceptional number of votes is that he truly enjoyed exceptional political support in those places. But there is no reason to believe that Buchanan enjoyed such mobilized support in Palm Beach County.

The 2000 vote for Buchanan in Palm Beach County was extremely unusual—clearly among the most unusual in the entire country. The vast difference between the residual for Palm Beach County and the residuals for the other counties confirms that a large anomaly occurred in Palm Beach County relative to the vote we would expect Buchanan to have received based on the voting history and demographic profile of the county. This county-level analysis does not provide direct evidence that the butterfly ballot is to blame. Starting with Section 3, we provide direct evidence for the butterfly ballot hypothesis. But first we shall show that the Reform vote in Palm Beach County was not anomalous in 1996.

The 1996 presidential ballot in Palm Beach County did not have a butterfly format. An excessively high number of Reform votes in 1996, compared to other Florida counties, could provide an alternative explanation for the apparently excessive Buchanan votes in 2000. Maybe support for Reform is just unusually high in Palm Beach County. On the other hand, if the Reform vote was exceptionally low in Palm Beach County in 1996, then the anomality of the 2000 vote could be exaggerated. The 2000 vote would appear to be excessive even if the county were simply returning to normalcy. The 1996 data support neither of these alternatives.

We analyze the count of votes received by the Reform candidate Ross Perot in the 1996 presidential elec-

tion across the counties of Florida. We apply the overdispersed binomial model to the number of votes cast for Perot out of all votes cast either for Perot, Democrat Bill Clinton or Republican Bob Dole. The regressors are defined to be earlier versions of the county-level variables we used to analyze the 2000 vote data: the proportion of votes officially received by the Republican candidate in the 1992 presidential election; the proportion of votes officially received by the Reform party candidate in the 1992 presidential election; and earlier demographic data.[31] We use our robust estimators.

The 1996 results contrast sharply with the 2000 findings. In 1996 no county in Florida has a residual even remotely as large as the one for Palm Beach County in 2000. In 1996 only St. Lucie County has a residual of absolute magnitude greater than 4.0 ($\tilde{r}_i = -4.92$). The largest positive residual is for Holmes County ($\tilde{r}_i = 2.30$). Palm Beach County has the seventh most negative residual ($\tilde{r}_i = -1.86$). Palm Beach County was not an outlier in 1996.

3 A Natural Experiment: Florida's Election-day and Absentee Voters in 2000

The previous section demonstrates that Buchanan's vote total in Palm Beach County in the 2000 presidential election was anomalously large and that this support neither was a reflection of historical support for the Reform party nor was consistent with the demographic composition of the county. It may be reasonable to conclude that the butterfly ballot caused this anomaly, but it is also conceivable that some other factor was responsible. For example, it could be that the county experienced an exceptional surge in voter support for either the Reform party in general or for Patrick Buchanan in particular. We now address this possibility.

Because the 2000 absentee presidential ballot in Palm Beach County was not a butterfly ballot, the election gives us a natural experiment: one group of Palm Beach County voters (election day) used a butterfly ballot but a second group (absentee) did not. If Buchanan's vote total in Palm Beach County reflects true support among the voters, then this support should be present in both pools of ballots. But if the butterfly ballot is responsible for Buchanan's vote then his support should have come disproportionately from election-day ballots.

A limitation of this natural experiment is that the mechanism that allocates voters to either the election-day pool or the absentee pool is not random assignment (Achen, 1986). Voters self-select to be in the

absentee pool.[32] Some Florida voters, e.g., military officers stationed overseas, must cast absentee ballots. Thus absentee voters may not be representative of voters in general. Absentee voters in Florida are generally thought to be more politically conservative than election-day voters. This means that the natural experiment that fosters a comparison of election-day and absentee ballots may be biased. Absentee voters may be more likely to vote for conservative candidates such as Bush or Buchanan than election-day voters are. If such a conservative bias does exist in Florida, it will make it more difficult to find that Buchanan received significantly greater support among election-day voters than absentee voters. That is, the bias would work against finding evidence for the butterfly ballot hypothesis. The potential for such a bias may be neutralized if absentee voters are more likely than election-day voters to choose Bush over Buchanan.

The differences between the election-day and absentee proportions of 2000 presidential votes that went to Buchanan show that there was disproportionate support for Buchanan among election-day voters in Palm Beach County. Figure 3 plots the differences—election-day proportion minus absentee proportion—for all 67 counties of Florida by the number of presidential ballots cast in each county.[33] One can see that Palm Beach County (which is labeled) has one of the largest differences in the state, albeit not the largest. Four counties have larger differences, and a few other counties have differences close in value to that of Palm Beach County. But all of those counties have voting populations much smaller than Palm Beach County's. In Palm Beach County 433,186 ballots were cast for president. In the largest of the counties that have differences larger than Palm Beach County there were 5,174 ballots cast for president.[34]

*** Figure 3 about here ***

The significance of the disparity in population sizes is that even if identical voting processes exist for each vote cast in all the counties, there will be greater variability in the differences of proportions in the counties that have smaller numbers of ballots cast. With identical processes in all counties, the standard deviation of the differences of proportions will vary in proportion to the reciprocal of the square root of the total number of ballots. If in all counties the proportion of election-day ballots cast for Buchanan is not systematically different from the proportion of absentee ballots cast for Buchanan, then the observed differences between the proportions should be contained within bounds defined by the reciprocal of the square root of the number of ballots.[35] In Figure 3 we have drawn dashed lines to indicate the location of the bounds. One can see that only Palm Beach County falls outside the bounds. The difference between the election-day and absentee proportions voting for Buchanan in Palm Beach County appears to be much

greater than one would expect by chance. The differences for the other counties appear to fall within the range one would expect given only random deviations from equality in the processes that generate votes by election-day and absentee voters.

The distorting effects of vastly different population sizes for counties may be fully corrected by explicitly setting up the comparison between election-day and absentee ballots as a test of the hypothesis that the proportion of votes for Buchanan among all the votes cast for president is equal for the two ballot forms. In order to compare the ballot forms we studentize the votes for Buchanan using the methods described in the Appendix (Section 9). As defined in the Appendix, z_{ai} is the studentized value for the absentee vote for Buchanan in county i and z_{bi} is the studentized value for the election-day vote for Buchanan. The difference between the studentized values is a good measure of whether the proportion of votes for Buchanan is equal across the two ballot forms. Under the equality hypothesis the difference $z_{bi} - z_{ai}$ is approximately normal with mean zero and variance equal to 2.0. If $z_{bi} - z_{ai}$ is significantly greater than zero, then in proportional terms election-day voters cast ballots for Buchanan much more often than absentee voters did. If $z_{bi} - z_{ai}$ is significantly less than zero, then support for Buchanan was disproportionately great among absentee voters.

Palm Beach County has $z_{bi} - z_{ai} = 4.8$, a difference that is more than three standard deviations away from the value of zero that is expected under the hypothesis of equality. The next largest positive value among the remaining counties of Florida is 1.36 and 62 counties have z-score differences of magnitude less than one. Only one other value, a difference of -1.74 for Duval County, is more than one standard deviation away from zero.

In Palm Beach County, election-day votes went to Buchanan vastly more often, in proportional terms, than absentee votes did. The results of the election-day versus absentee ballot natural experiment strongly support the conclusion that Buchanan's anomalous support was caused by the butterfly ballot.

Underlying the dramatic test statistic value for Palm Beach County is the fact that Palm Beach County election-day voters supported Buchanan at a rate approximately four times that of absentee voters: for Palm Beach County the proportion of Buchanan voters is .0085 among election-day voters but it is only .0022 among absentee voters. If election-day voters had cast ballots for Buchanan at the rate that absentee voters did, then Buchanan would have received $387356 \times 0.0022 \approx 854$ election-day votes. In fact he received 3,310 election-day votes. By this method one might gauge the number of accidental votes Buchanan received because of the butterfly ballot to be approximately $3310 - 854 = 2,456$ votes.

4 Precinct-Level Analysis of Palm Beach County Returns

The results so far strongly suggest that Buchanan's vote total in Palm Beach County was not a true reflection of voter intentions. A crucial question remains: Who made the mistakes? Was it voters who wanted to vote for Bush or those who wanted to vote for Gore?

At an intuitive level, mistakes seem less likely for Bush voters, who had to match the first candidate with the first punch hole, than for Gore voters, who had to match the second candidate with the third punch hole. Furthermore, it was a large number of Democratic voters who complained that the butterfly ballot caused them to vote for Buchanan by mistake. Nonetheless it is important to examine the possibility that Buchanan received votes intended for both candidates.[36]

The analysis in this section is intended to assess the asymmetry of voters' errors: whether Democratic voters mistakenly chose Buchanan at a substantially greater rate than Republican voters did. To do that, we estimate an overdispersed binomial model for precinct-level election returns across Palm Beach County. We use two regressors (plus the constant): the proportions of the vote in each election-day precinct for U.S. Senate candidates Democrat Bill Nelson and Reform candidate Joel Deckard. We use our robust estimators. If the butterfly ballot did not cause systematically biased voting errors, we would expect that support for Nelson (D) in a precinct would be *negatively* associated with support for Buchanan and that support for Deckard (Ref) would be *positively* associated with support for Buchanan. If the butterfly ballot did cause asymmetric voting errors with Democrats inadvertently voting for Buchanan, we would expect support for Nelson (D) to be *positively*—instead of *negatively*—associated with support for Buchanan.

The first line in Table 2 presents the results for election-day precincts across the whole of Palm Beach County. Support for Nelson (D) is *positively* associated with support for Buchanan. This pattern is what we would expect if the butterfly ballot caused many Democrats to vote for Buchanan. The pattern supports the claim that Buchanan's votes tended to come from mistaken Gore supporters.

*** Table 2 about here ***

The pattern does not occur for votes for other offices that included a Reform candidate but did not use the butterfly ballot. In 2000 in Palm Beach County, Sherree Lowe (Ref) ran for the State Senate in District 35 and John McGuire (Ref) ran for the U.S. House in District 16. Because only the presidential portion of the 2000 Palm Beach County ballot used the butterfly format, we are able to examine the relationship between Reform party support and Democratic party support in Palm Beach County both in the presence

and in the absence of the butterfly ballot. The issue positions stated on the candidates' web pages suggest that all three of the Florida Reform candidates are in sympathy with the Buchanan faction of Reform. For instance, all are pro-life on the abortion issue and all look askance at free trade. Deckard goes so far as to express concerns about *legal* immigration.[37]

We robustly estimate two models for election-day precincts in Palm Beach County in each district. In both districts we analyze the votes received by Buchanan. For District 35 we also analyze the votes received by Lowe (Ref) and for District 16 we analyze the votes for McGuire (Ref). Because the butterfly ballot was relevant only for the presidential vote, we expect support for Nelson (D) to be *negatively* associated with support both for Lowe (Ref) and for McGuire (Ref).

The results in Table 2 match those expectations, while in both districts support for Nelson (D) remains *positively* associated with support for Buchanan. We also find that support for Deckard (Ref) is not significantly associated with support for Buchanan in either district, but Deckard support is positively associated with support for McGuire (Ref).

In the absence of claims about the butterfly ballot, the precinct-level findings would be quite counterintuitive. One would expect precincts that are *more* Democratically inclined with respect to the U.S. Senate race to be *less* Reform-inclined when it comes to other races. The results presented in Table 2 support the claim that the butterfly ballot caused systematic, biased voter errors that cost Gore more lost votes than Bush. Democratically inclined precincts (as measured by the Nelson (D) vote proportion) have fewer votes for Reform candidates in general (i.e., Lowe and McGuire) but have more votes for Buchanan. The difference is the butterfly ballot.

The key features of Table 2 are the contrasts between Buchanan and Lowe and between Buchanan and McGuire. It is possible to explain the positive association between Nelson's (D) vote share and votes for Buchanan by asserting that Reform members in Palm Beach County tend to live among Democrats. The unintuitive nature of such an assertion notwithstanding, the assertion contradicts the finding that the proportion of votes for Nelson (D) is negatively associated with the votes both for Lowe (Ref) and McGuire (Ref). Also there is no significant relationship between votes for Deckard, the Reform candidate for the U.S. Senate, and votes for Buchanan. But there is, as one would expect, a strong positive relationship between votes for Deckard and votes for at least one of the other two Reform candidates.

The results in this section also enable us to examine—and refute—one final class of alternative ex-

planations for Buchanan's anomalous vote total in Palm Beach County. It is conceivable that Buchanan's Palm Beach County vote total was caused by a group of anomalous precincts within the county. Anomalous results concentrated within a few precincts would suggest that excess votes were the result of localized phenomena, rather than the butterfly ballot, which was used uniformly throughout the county. For example malfunctioning vote machines—like the one in Tillman precinct in Jasper County, South Carolina—could have recorded extra votes for Buchanan in a few precincts. Alternatively, intentional fraud in a few precincts could be the source of Buchanan's success. Finally, pockets of intense election-day Reform mobilization could have delivered the extra votes.

Such explanations are, however, quite difficult to reconcile with the precinct-level patterns documented in this section. Given our use of a robust estimator, the coefficient estimates in Table 2 would not be affected by a few precinct-level anomalies produced by irregular voting processes.[38] Moreover, a localized mobilization effort should affect outcomes in multiple races whereas the peculiar relationship between Nelson (D) vote share and Reform vote is present only in the presidential race, which used the butterfly ballot.[39]

5 Comparisons Using Individual Palm Beach County Ballots

We extend our analysis to ballot-level data from Palm Beach County.[40] The data include both election-day and absentee ballots but exclude returns for 25 precincts that were overwritten by test data when Palm Beach County tested their vote tabulating machines. The number of Buchanan votes lost from those precincts (2.1 percent) is proportional to the number of precincts lost. The ballot data enable us to compare an individual's presidential vote choice with the individual's choices for other offices. We use the ballot-level data to validate the precinct-level regression results. We also produce another estimate of the size of the butterfly ballot effect.

With ballot-level data the binomial regression model reduces to the familiar binary logistic regression model.[41] In addition to the constant, the regressors are two dummy variables that respectively indicate whether the ballot records a vote for Nelson (D) or a vote for Deckard (Ref).

The estimates reported for election-day ballots in Table 3 show a positive and significant coefficient on voting for Nelson (D), matching the positive coefficient on the precinct-level proportion of votes for Nelson (D) that appears in Table 2. Among election-day ballots, voting for the Democratic candidate for the U.S. Senate (Nelson) is *positively* associated with voting for Buchanan for president. That is not the case with

the absentee ballot. Among the absentee ballots the Nelson (D) coefficient is negative and insignificant. The confidence intervals of the Nelson (D) coefficients for the two ballot forms do not overlap. Hence we may reject the hypothesis that a Nelson (D) voter has the same chance of voting for Buchanan regardless of the form of the ballot.

*** Table 3 about here ***

The proportions in Table 4 show that Palm Beach County voters who support the Democratic Senate candidate are significantly more likely to vote for Buchanan on the butterfly ballot than are their counterparts who use the absentee ballot. The table lists the proportion of votes in Palm Beach County going to Buchanan among all ballots that record U.S. Senate votes for either Nelson (D) or Deckard (Ref). Individuals who vote for the Democratic Senate candidate are six times more likely to vote for Buchanan using the butterfly ballot than the absentee ballot. Fewer than two in a thousand absentee voters in Palm Beach County who vote for Nelson (D) also vote for Buchanan, while the proportion of election-day Nelson (D) voters who also vote for Buchanan is ten in a thousand. If we treat the absentee proportion as the proportion of votes truly intended to go to Buchanan, then about 8.5 of every 1,000 Nelson (D) voters in Palm Beach County—about 2,300 voters—appear to have mistakenly voted for Buchanan.[42]

*** Table 4 about here ***

Table 4 shows that individuals who vote for Deckard (Ref) are more likely to vote for Buchanan using the absentee ballot. Deckard voters who support Buchanan should not be affected by the butterfly ballot, and the difference between election day and absentee Buchanan vote proportions is small.

The ballot data add to the evidence that the butterfly ballot caused systematic voting errors in Palm Beach County that cost Gore votes. In particular the ballot data help rule out the possibility that Buchanan's exceptional support in the county was a result of populist appeals he made or policy positions he took that Democrats found attractive. Such an explanation is already implausible because there is no reason to believe such appeals should have worked so much better for Buchanan in Palm Beach County than they did anywhere else in Florida. Certainly, according to Bay Buchanan, he made no special effort to communicate his views to voters there.[43] But the ballot data show further that the appeals would somehow have to have been effective for Democrats who voted on election day but not Democrats who used an absentee ballot.

We gain analytical precision with the ballot data, but this section complements rather than replaces the county and precinct analyses presented above. Ballot-level data are rarely retained or made available after an

election, and therefore it is not generally possible to compare these results across states or counties. Without such comparisons, the ballot-level results must be considered with some caution. Moreover, data from two percent of the precincts in Palm Beach County ballot are not available. Our precinct-level analysis does not suffer from missing data. Also, unlike the county-level and precinct-level analysis, the analysis of individual ballots cannot use robust estimation techniques. In the absence of robust—high breakdown point—results, it would have been possible to claim that the aberrations we have found may be limited to a few idiosyncratic precincts and not characteristic of Palm Beach County as a whole. But the aberrations prevail throughout the county.

6 Conclusion

We have examined the source of the anomalous support for Buchanan in Palm Beach County by focusing on allegations that the county's use of a butterfly ballot caused systematic voting errors that boosted the number of votes for Buchanan.[44] In particular, we used robust estimation of overdispersed binomial regression models to show that, with respect to the Reform vote in 2000, Palm Beach County is the largest outlier among all counties in the United States we are able to examine. We also showed that Palm Beach County was not a Reform vote outlier in 1996, the year of a presidential election in which the county did not use a butterfly ballot.

Having confirmed that in 2000 Palm Beach County was an outlier, we sought to verify whether the butterfly ballot was indeed the cause. A comparison of election-day versus absentee ballot results across all of Florida's counties shows that Buchanan's success in Palm Beach County did not extend to Palm Beach County's absentee voters, who did not use the butterfly ballot. We have examined the claim that Democratic presidential candidate Al Gore in particular was harmed by the butterfly ballot. We found that Buchanan's support in Palm Beach County tended to come from more Democratic precincts and from voters who voted for the Democratic candidate for the U.S. Senate, a pattern that supports the claim that Buchanan's votes tended to come from mistaken Gore supporters.

Was the butterfly ballot pivotal in the 2000 presidential race? The evidence is very strong that it was. Had Palm Beach County used a ballot format in the presidential race that did not lead to systematic biased voting errors, our findings suggest that, other things equal, Al Gore would have won a majority of the officially certified votes in Florida.

Our analysis complements the efforts media groups have been making to inspect the ballots throughout Florida in order to assess what result would have been produced by completing the recount that the U.S. Supreme Court terminated, or by conducting a count using uniform standards throughout the state. As of this writing only the results of a statewide inspection conducted by the Miami Herald have been reported.[45] Our analysis answers a counterfactual question about voters' intentions that such inspection efforts cannot resolve. The inspections may clarify the number of voters who marked their ballots in support of the various candidates, but the inspections cannot tell us how many voters marked their ballots for a candidate other than the one they intended to choose.

Citing the results from various scenarios in which votes were counted using one of several reasonable uniform standards, the Herald concludes, "After study and analysis of 111,261 overvotes and 64,826 undervotes, [...] the outcome still depends on the standard used to gauge undervotes. Gore wins narrowly under two undervote standards, by margins of 332 and 242 votes; Bush wins narrowly under two other undervote standards, by 407 and 152 votes."[46] Evidently the number of votes that were intended for Gore but that went to Buchanan, because of the butterfly ballot, is large enough to have changed the outcome of the election given any of several reasonable standards that might have been used to count the votes in Florida.

Although we have focused here on the butterfly ballot in Palm Beach County, our methods could be used on a regular basis as part of an ongoing effort to identify election anomalies and to develop improvements in the administration of elections to help eliminate such anomalies. Our robust estimation and outlier detection methods offer an accurate and powerful technology for detecting irregular vote outcomes. But determining *why* a given irregular outcome occurred requires a strategy of triangulation, such as the one we have pursued here. Different models and different types of data need to be marshaled to eliminate plausible alternative explanations. In the case of Palm Beach County and the 2000 presidential election, such a strategy leads to the conclusion that "The Butterfly Did It."

7 Appendix: Estimation Methodology

7.1 The Rationale for Robust Estimation

The primary reason to use robust estimators is that the voter complaints, legal cases and media reports strongly suggest that the processes that produced the electoral results in Palm Beach County were sub-

stantially different from the standard political factors (party identification, liberalism-conservatism, policy positions) that produced the results elsewhere in Florida. We do not want our parameter estimates to be distorted by idiosyncratic factors that cause deviations from the standard political factors that cause voters to act predictably from one election to another. An outlier may have a different disturbance (e.g., a nonzero mean, a fat-tailed distribution), different regression coefficients or contaminated regressors.

Another reason to use robust estimators is the fact that the regression models we use are at best rough approximations for the processes that produced the vote counts (Hampel, Ronchetti, Rousseeuw, and Stahel, 1986, 82). The estimators we use produce reliable measures of discrepancy even under such conditions, as long as the model gives a pretty good approximation for Buchanan's expected vote in most counties in each state.

Data weakness is another reason for robust estimation. Because our models are based on the results from the previous election, anomalies in that election will give a distorted picture of the standard political factors that predict the vote in the current one. The robust estimators we use protect against the influence such distortions might otherwise have on the results. A county whose previous election results are highly distorted will not affect the results for the other counties and will itself appear as an observation that has a large discrepancy.

Because the robust estimators we use have a high *breakdown point* (Hampel, 1971; Donoho and Huber, 1983), they are consistent and produce reliable measures of discrepancy even if unusual voting processes occurred in several counties in a state.[47] A large anomaly in one county will not mask (Atkinson, 1986) comparable or perhaps somewhat smaller anomalies that occur in other counties (Hampel et al., 1986, 67). An estimator that lacks a high breakdown point will underestimate the frequency of highly anomalous election results. The robust methods we use also perform well in the absence of anomalies. If there are no anomalies, the robust estimator remains consistent and is almost as efficient as an estimator such as simple iteratively reweighted least squares that ignores the possibility of anomalous observations.

7.2 The County-level Overdispersed Binomial Regression Model

For the results of the county-level analysis that we report in detail in the text we use only the first principal component of the census data. As we explain in the text, using more principal components does not substantively change the results that bear on Palm Beach County. Hence the expected vote for Buchanan in county

i in state s is based on a linear predictor defined by:

$$x_i'\beta_s = \beta_{s0} + \beta_{s1}x_{1i} + \beta_{s2}x_{2i} + \beta_{s3}x_{3i} \tag{1}$$

where x_{1i} is the 1996 proportion of votes received by the Republican candidate, x_{2i} is the 1996 proportion of votes received by the Reform candidate, x_{3i} is the principal component and β_{s0}, β_{s1}, β_{s2} and β_{s3} are constant coefficients to be estimated.

We model the number of votes cast for Buchanan out of all votes cast for either Buchanan, Gore, Bush, Ralph Nader (Green), Harry Browne (Libertarian), Howard Phillips (Constitution), John Hagelin (Natural Law) and any write-in candidates. Because the dependent variable records counts we use binomial models. Following McCullagh and Nelder (1989, 125), we allow for overdispersion because we believe that the county-level data are subject to unobserved internal clustering effects.

The overdispersed binomial model is defined as follows. Let i indicate one of the n_s counties in state s, $i = 1, \ldots, n_s$. Given a vector of k regressors x_i and given the total number m_i of ballots cast for presidential candidates in county i, the count y_i of votes for Buchanan has expected value:

$$E(y_i \mid x_i, m_i) = m_i\pi_i, \tag{2}$$

with probability π_i being a logistic function of the linear predictor $x_i'\beta_s$ specified in (1):

$$\pi_i = \frac{1}{1+\exp(-x_i'\beta_s)}, \tag{3}$$

where β_s is an unknown constant vector of coefficient parameters. The model departs from a standard binomial because the variance is:

$$E[(y_i - m_i\pi_i)^2 \mid x_i, m_i] = \sigma_s^2 m_i\pi_i(1-\pi_i), \tag{4}$$

with $\sigma_s^2 > 0$ (McCullagh and Nelder, 1989, 125, eqn. 4.20). If $\sigma_s^2 > 1$ then there is overdispersion relative to a purely binomial model.

As discussed in sections 7.3 and 7.4, we compute a LQD (Croux, Rousseeuw, and Hossjer, 1994; Rousseeuw and Croux, 1993) estimate of the scale $\sigma_s = \sqrt{\sigma_s^2}$ because the LQD is robust and reasonably

efficient. Let $\hat{\sigma}_s$ denote the estimated scale value obtained with the LQD estimator. Given $\hat{\sigma}_s$, we compute a tanh (Hampel et al., 1981) estimate for β_s. Let $\hat{\beta}_s$ denote the estimated coefficient vector obtained with the tanh estimator. As section 7.4 further explains, once we have a reasonable estimate of scale such as the one produced by an LQD estimator, the tanh estimator of the coefficients has additional optimality properties. The tanh estimation provides a weight $w_i \in [0,1]$ for each county (see equation (13)) that provides information about whether the county is an outlier. If $w_i = 0$ then county i is an outlier and, given $\hat{\sigma}_s$, data from the county is not used in forming the estimate $\hat{\beta}_s$.

Given expected proportions $\hat{\pi}_i = [1+\exp(-x_i'\hat{\beta}_s)]^{-1}$ of votes for Buchanan, we use studentized residuals to measure the discrepancy between actual and expected votes for Buchanan. The studentized residual is:

$$\tilde{r}_i = r_i/\sqrt{1-h_i}. \tag{5}$$

where:

$$r_i = \frac{y_i - m_i\hat{\pi}_i}{\hat{\sigma}_s\sqrt{m_i\hat{\pi}_i(1-\hat{\pi}_i)}} \tag{6}$$

and h_i adjusts for leverage (applying to the counties that have $w_i > 0$) or for forecasting error (applying to the counties that have $w_i = 0$). Let W be a diagonal matrix that has diagonal entries $W_{ii} = w_i$, V be a diagonal matrix with $V_{ii} = [m_i\hat{\pi}_i(1-\hat{\pi}_i)]^{-1/2}$, and X be the $n \times k$ matrix of the regressors (row i of X is x_i'). Using the diagonal values of $H = VX(X'VWVX)^{-1}X'V$ we obtain the robust estimates $h_i = H_{ii}$ if $w_i > 0$ and $h_i = -H_{ii}$ if $w_i = 0$. McCullagh and Nelder (1989, 397) motivate a nonrobust version of H. The studentized residuals may be compared across counties, both within and across states.[48]

To summarize, the innovations in our methods are: use of LQD to estimate the overdispersed binomial model's dispersion parameter; and use of the tanh estimator for the overdispersed binomial model's coefficients. As far as we know, the robust methods have not previously been applied in these ways.

7.3 Some Properties of Robust Estimation Methods

For intuition about some basic properties of robust estimation methods, consider the limitations of classic least squares. The linear regression model for n observations and k regressors has the form $y_i = x_i'\gamma + \varepsilon_i$, $i = 1,\ldots,n$, where γ is a vector of k unknown coefficients, the data y_i and x_i are observed, and the unobserved

disturbance ε_i has conditional mean $E(\varepsilon_i \mid x_i) = 0$ and variance τ^2. The goal is parameter estimates $\hat{\gamma}$ and $\hat{\tau}^2$ that converge in probability to γ and τ^2 as n gets large: we want *consistent* estimates.[49] Least squares (LS) chooses $\hat{\gamma}$ to minimize the sum of squared residuals $u_i = y_i - x_i\hat{\gamma}$ over all $i = 1, \ldots, n$:

$$\hat{\gamma}_{\text{LS}} = \underset{\hat{\gamma}}{\text{argmin}} \sum_{i=1}^{n} u_i^2. \tag{7}$$

The LS estimate for τ^2 is then the mean (adjusted for the number of coefficients) of the squared residuals $(y_i - x_i\hat{\gamma}_{\text{LS}})^2$. Even one contaminated data point can cause $\hat{\gamma}_{\text{LS}}$ to take values arbitrarily different from γ. Hence LS has a breakdown point of $1/n$ (asymptotically, 0). LS is not robust.

For the regression model the maximum possible breakdown point is, asymptotically, 0.5. One popular estimator for γ that achieves that maximum is *least median of squares* (LMS):

$$\hat{\gamma}_{\text{LMS}} = \underset{\hat{\gamma}}{\text{argmin}} \underset{i}{\text{med}}\, u_i^2 \tag{8}$$

where $\text{med}_i\, u_i^2$ denotes the median of the u_i^2 values, $i = 1, \ldots, n$ (Rousseeuw, 1984; Rousseeuw and Leroy, 1987). LMS has two important defects. $\hat{\gamma}_{\text{LMS}}$ is consistent for γ, but the estimator converges at the slow rate of $n^{-1/3}$. LMS also is inefficient when the disturbance ε_i is an identically and independently distributed Gaussian random variable (i.e., no ε_i outliers) and the model is otherwise correctly specified. One way to achieve greater efficiency is to use LMS estimates as starting values for a redescending *M*-estimator such as the tanh estimators to be described in the next subsection (Hampel et al., 1981).

Other estimators exist that achieve the maximum breakdown point while having a $n^{-1/2}$ convergence rate and better Gaussian efficiency than LMS does. The LQD estimator for γ is such an estimator (Croux et al., 1994). LQD has a Gaussian efficiency of 67.1% for all the elements of γ except the intercept, which LQD does not estimate in a linear regression model (Croux et al., 1994). In addition to an efficiency gain over LMS, LQD provides a superior estimate of the scale (i.e., τ) when ε_i has an asymmetric distribution, because LQD does not estimate the scale by measuring a symmetric spread of the residuals around a central location value (Rousseeuw and Croux, 1993).

7.4 Robust Overdispersed Binomial Model Implementation

The first step in the robust estimation of the overdispersed binomial model is to get an estimate of the dispersion parameter σ_s^2 that appears in equation (4) above. To use LQD to estimate σ^2 we need to define residuals that are reasonably well described by a reference model of normality with zero mean and variance σ_s^2. If the overdispersed binomial model is correctly specified then given a consistent estimate $\hat{\beta}_s$ for β_s, residuals of the form:

$$r_i^* = \frac{y_i - m_i\hat{\pi}_i}{\sqrt{m_i\hat{\pi}_i(1-\hat{\pi}_i)}} \tag{9}$$

are approximately normal with the desired mean and variance. Indeed, in the absence of outliers a good moment estimator for σ_s^2 may be defined in terms of r_i^* (McCullagh and Nelder, 1989, 126–127, eqn 4.23).

The approximate normality of r_i^* depends on conditions such as independence across i and sufficiently large values for $m_i\pi_i(1-\pi_i)$. In view of the small proportion of the votes that Buchanan received, it is important to consider for how small a value of $m_i\pi_i$ the normality model is plausible. Notice that $X^2 = \sum_{i=1}^{n}(r_i^*)^2$ is the Pearson chi-squared statistic. Larntz (1978, 255–256) enumerates the exact distribution of X^2 for binomial data and concludes that, even for sample size n as small as 10 or 15, inferences reached by using the asymptotic chi-squared distribution are reasonable when all expected values $m_i\pi_i$ and $m_i(1-\pi_i)$ are greater than 1.0. We take such results as supporting our use of the reference normal model for the county data. No state has more than a few counties that have m_i so small that the expected vote for Buchanan is less than 1.0. The case is less clear for the precinct data. Because the expected proportion of the vote for Buchanan in Palm Beach County is about 0.001 (see Table 5), a typical precinct would have to have $m_i = 1,000$ voters to have one expected vote for Buchanan. Among the 515 election-day precincts in Palm Beach County, 172 have $m_i > 1000$ but 56 have $m_i < 100$. Koehler and Larntz (1980, 337) show (for Poisson variables) that the Poisson information kernel declines rapidly to zero as the expected value goes from one to zero. Koehler and Larntz (1980, 338) observe that consequently the asymptotic (in n) mean of the likelihood-ratio chi-squared statistic "can be much smaller than the chi-squared mean when many expected frequencies are smaller than one-half"; McCullagh (1986) demonstrates one important respect in which the chi-squared approximation survives better for the Pearson statistic than for the likelihood-ratio statistic as m_i decreases to one. That may suggest that our estimates of σ are biased somewhat downward

for the precinct data.

Because of the possibilities of outliers, we need a robust estimate of σ_s^2. One may understand the LQD estimator intuitively as an extension of the idea of using the interquartile range to estimate the dispersion of a set of data. The LQD estimator focuses on the $\binom{h_k}{2}$ order statistic of the set $\{|r_i^* - r_j^*| : i < j\}$ of absolute differences, where $\{|r_i^* - r_j^*| : i < j\}$ has $\binom{n}{2}$ elements and $h_k = \lceil (n+k)/2 \rceil$. Following Croux et al. (1994) we use the notation

$$Q_n^* = \{|r_i^* - r_j^*| : i < j\}_{\binom{h_k}{2}:\binom{n}{2}} \tag{10}$$

to denote that order statistic. To implement LQD we choose estimates $\hat{\beta}_s$ to minimize Q_n^*.[50] Let $\hat{\beta}_{\text{LQD}*}$ designate the estimated coefficient vector and let $\hat{Q}_n^*$ designate the corresponding minimized value of Q_n^*. The LQD scale estimate is

$$\hat{\sigma} = \hat{Q}_n^* \frac{1}{\sqrt{2}\Phi^{-1}(5/8)}, \tag{11}$$

where Φ^{-1} is the quantile function for the standard normal distribution (Rousseeuw and Croux, 1993, 1277). For large n and k small relative to n, $\binom{h_k}{2}/\binom{n}{2} \approx 1/4$ so that Q_n^* is approximately the first quartile of the absolute differences.

The LQD objective function is difficult to optimize. Because high breakdown-point estimators are not smooth functions of the data, optimization typically does not rely on derivative information but instead depends on combinatorial algorithms (Stromberg, 1993). We use the global optimizer GENetic Optimization Using Derivatives (GENOUD) (Sekhon and Mebane, 1998). GENOUD combines evolutionary algorithm methods with a derivative-based, quasi-Newton method to solve difficult unconstrained optimization problems.[51]

Because the LQD estimator provides an estimate of β_s and because it is better than LMS, we could use it to find outliers. But to minimize the chances of falsely identifying counties as outliers we would like to improve further. To do that we use a tanh estimator for β_s that uses the LQD scale estimate. The tanh estimator is a one-dimensional redescending *M*-estimator of location for which Hampel et al. (1981) prove existence and optimality properties (see also Hampel et al., 1986, 160–168). In addition to achieving the maximum breakdown point, tanh estimators minimize the asymptotic variance of the estimates for a given

upper bound d on how sensitive the variance is to a change in the distribution of the data (Hampel et al., 1986, 125–136). Given a good estimate of the scale, then, tanh estimators are the most efficient possible estimators of location that have the key robustness property of having bounded response to arbitrary changes in parts of the data (Hampel et al., 1986, 166).

The tanh estimator (Hampel et al., 1981) is based on the function

$$\psi(u) = \begin{cases} u, & \text{for } 0 \leq |u| \leq p \\ (A(d-1))^{1/2} \tanh[\frac{1}{2}((d-1)B^2/A)^{1/2}(c-|u|)] \operatorname{sign}(u), & \text{for } p \leq |u| \leq c \\ 0, & \text{for } c \leq |u| \end{cases} \tag{12}$$

where p, c, d, A and B are constants satisfying $0 < p < c$ and other conditions.[52] For $c > |u|$, the sign of $\psi(u)$ equals the sign of u. For u increasing from p to c, $\psi(u)$ descends from its maximum value $\psi(p) = p$ to $\psi(c) = 0$, and for u increasing from $-c$ to $-p$, $\psi(u)$ descends from $\psi(-c) = 0$ to its minimum value $\psi(-p) = -p$. Hampel et al. (1981, 645) and Hampel et al. (1986, 162) depict the shape of ψ. Optimality requires that along the curves between $\psi(p)$ and $\psi(c)$ and between $\psi(-c)$ and $\psi(-p)$, the value of ψ is such that the ratio of the sensitivity of the variance to a change in the data over the asymptotic variance of the data is a constant. Indeed, the value of the ratio is d. Given a choice of d and of the truncation point c, minimizing the asymptotic variance of the estimator while satisfying such conditions implies unique values for p, A and B of (12) (Hampel et al., 1986, 162–164). We use $c = 4.0$ and $d = 5.0$ which imply values $p = 1.8$, $A = 0.86$ and $B = 0.91$ (Hampel et al., 1981, 645, Table 2).[53]

Given a scale estimate $\hat{\sigma}_s$ and a vector of trial estimates $\hat{\beta}_s$, we compute the residuals r_i of (6) and then weights

$$w_i = \begin{cases} \psi(r_i)/r_i, & \text{for } r_i \neq 0 \\ 1, & \text{for } r_i = 0. \end{cases} \tag{13}$$

Observation i is weighted by w_i in what is otherwise the usual iteratively reweighted least squares algorithm to estimate β_s. The value of c fixes the threshold for the magnitude of r_i beyond which an observation is completely rejected by assigning it a weight $w_i = 0$. The scale value $\hat{\sigma}_s$ remains unchanged but the weights are updated to match the current coefficient estimates. Numerical convergence is required for both the $\hat{\beta}_s$

values and the weights.[54] Because redescending M-estimators such as the tanh estimator have multiple solutions, starting values affect the results. We use $\hat{\beta}_{\mathrm{LQD}*}$ to start the coefficients and use the LQD values $(r_i^* - \mathrm{med}_i\, r_i^*)/\hat{\sigma}_s$ for an initial set of residuals r_i.

To estimate the asymptotic variance of the coefficient estimates (and hence the standard errors reported in Table 2), we use the sandwich estimator $\mathrm{avar}(\hat{\beta}) = \hat{\sigma}^2 \hat{J}^{-1} \hat{I} \hat{J}^{-1}$ where $\hat{I}$ is the outer product of the score and $\hat{J}$ is the Hessian for a standard binomial likelihood evaluated at $\hat{\beta}$, weighting each observation by w_i. The sandwich estimator is valid insofar as the conditions for Theorem 6.4 of White (1994, 92) hold. The necessary assumptions are 2.1 (complete probability space), 2.3 (measurability, compact parameter space and continuity), 3.1 (uniform convergence), 3.2' (interior identifiably unique maximizers), 3.6 (continuous differentiability), 3.7(a) (uniform convergence of score vector), 3.8 (uniform convergence of Hessian), 3.9 (negative definite Hessian) and 6.1 (score obeys central limit theorem with positive definite covariance matrix). Hampel et al. (1986, 82) directly assume 2.1 and 2.3. The assumption of Fisher consistency (Hampel et al., 1986, 83) implies 3.1. Given a suitable model parameterization, their Theorem 5 (Hampel et al., 1986, 160–162) implies 3.6 and 3.7(b) for all observations that are not outliers, and optimization by Newton's method from suitable starting values (compare Hampel et al. (1986, 152)) implies 3.2', 3.8, 3.9 and 6.1.

8 Appendix: County-level Election Returns Data Sources and Table of Outliers

For AL, AR, AZ, CO, CT, GA, IA, ID, IL, IN, KS, KY, LA, MA, MD, MN, MO, MS, MT, ND, NE, NH, NJ, NM, NV, NY, OH, OR, PA, RI, SC, SD, TN, TX, UT, VA, WA and WI data are certified (or "official") election results from each state's Secretary of State (or comparable office), or results reported on the Secretary of State's website, originally collected by David Leip (with updates as of December 16, 2000) and posted at `http://www.uselectionatlas.org` (accessed January 10 or 21, 2001). For NY we obtained data separately for the City of New York.[55] For ME (December 18, 2000) and WV (December 20, 2000) we accessed data from `http://www.uselectionatlas.org` on the indicated dates. For the remaining states except AK we obtained data from official websites, as follows: CA, California Secretary of State, Election 2000[56]; DE, Secretary of State, Department of Elections[57]; FL, Florida Department of State, Division of

Election, "Data Download Utility"[58]; NC, North Carolina State Board of Elections, Official Results[59]; OK, Election Results and Statistics, 2000, Oklahoma State Election Board[60]; VT, 2000 Vermont Election Results, State of Vermont, Office of the Secretary of State, Elections & Campaign Finance Division[61]; WY, 2000 Official Election Results, Wyoming Secretary of State, Election Administration[62]. For AK, we use the 25 county equivalents that were in effect during the 1996 election, leaving Denali Borough within Yukon-Koyukuk Census Area, and Yakutat City and Borough within Skagway-Hoonah-Angoon Census Area. We aggregate precinct vote data to create the county-equivalent units. Precinct-level results for 2000 are in pdf files dated December 5, 2000[63] and for 1996 are in text files dated November 27, 1996.[64] To map the data into county equivalents we used voting district and state legislative district data provided by the Census Bureau.[65]

*** Table 5 about here ***

9 Appendix: Derivation of the Natural Experiment Test Statistics

Under the equality hypothesis, the best measure of the proportion of votes for Buchanan is the proportion voting for him among all the votes cast for president. Let A_i denote the total number of absentee votes cast for president in county i and let a_i denote the number of absentee votes cast for Buchanan. Let B_i denote the total number of election-day votes cast for president and let b_i denote the number cast for Buchanan. Using p_i to denote the proportion for Buchanan among all the votes for president in county i, we have $p_i = (a_i + b_i)/(A_i + B_i)$. Consistent with Section 2, we use an overdispersed binomial model for the totals a_i and b_i, with statewide dispersion parameter σ^2. Under the equality hypothesis the expected number of absentee votes for Buchanan is $A_i p_i$ and the expected number of election-day votes for Buchanan is $B_i p_i$. The variance of a_i is $\sigma^2 A_i p_i (1-p_i)$ and the variance of b_i is $\sigma^2 B_i p_i (1-p_i)$. Under the equality hypothesis and using the overdispersed binomial model, each of the following z-scores is approximately normal with mean zero and unit variance:

$$z_{ai} = \frac{a_i - A_i p_i}{\sigma\sqrt{A_i p_i (1-p_i)}} \tag{14a}$$

$$z_{bi} = \frac{b_i - B_i p_i}{\sigma\sqrt{B_i p_i (1-p_i)}}. \tag{14b}$$

To compute the z-scores we need an estimate of σ. We use the estimate of the scale parameter that we obtained for the 2000 Florida data when we estimated the county-level overdispersed binomial model. That estimate is $\hat{\sigma}_s = 3.81$.

Notes

** A previous version of this paper was presented at the 2001 Annual Meeting of the Public Choice Society, San Antonio, Texas, March 9–11. The authors, who are listed in reverse alphabetical order, served as unpaid expert witnesses in Case CL 00-10992AF in the Fifteenth Judicial Circuit of Florida, West Palm Beach: Beverly Rogers and Ray Kaplan vs. The Palm Beach County Elections Canvassing Commission; et al. (see Brady, Herron, Mebane, Sekhon, Shotts, and Wand, 2001). Along with other similar cases, the case was dismissed as a matter of law by the Supreme Court of Florida (Case Number SC 00-2373, Fladell, et al. vs. Palm Beach County Canvassing Board, etc. et al.). The authors thank Todd Rice and Lamarck, Inc. for their generous support and provision of computing resources, Kevin Matthews of the Department of Geography, George Mason University, for mapmaking services, Benjamin Bishin and Laurel Elms for research assistance, and seminar participants at Columbia University, Cornell University, Harvard University, and Northwestern University for helpful comments.

[1]"Voting Scrutinized All Over Florida," *Associated Press*, Thursday, November 9, 2000. Don Van Natta Jr. and Michael Moss, "Democratic 'War Room' Tries to Oversee the Battle for Florida, to Mixed Results," *New York Times*, Saturday, November 11, 2000. David Firestone, "Party Mobilized on Ballot Even as Votes Were Cast," *New York Times*, Saturday, November 11, 2000. David Firestone, "Democrats Still Prefer to Keep Counting," *New York Times*, Saturday, November 11, 2000. Don Van Natta Jr., "Gore to Contest Recount Result in Palm Beach," *New York Times*, Saturday, November 25, 2000.

[2]The suits filed by such voters in the Fifteenth Judicial Circuit of Florida, West Palm Beach, were cases CL 00-10965, CL 00-10970, CL 00-10988AE, CL 00-109922AF, CL 00-11000AH, CL 00-11084AH, CL 00-11098AO, CL 00-1146AB, CL 00-1240AB, CL 00-129OAB and CL 00-113O2AO. The suits were consolidated by Administrative Order No. 2.061-11/00. Texts of the filings and of the Circuit Court rulings in the cases are available from `http://www.pbcountyclerk.com/`.

[3]Bush received 271 electoral votes, one more than the 270 needed to win, while Gore received 266 electoral votes. One Elector who was pledged to Gore from Washington, D.C., left her ballot blank, hence reducing Gore's count from 267. David Stout, "As Electors Cast Their Votes, Bush Confers in Washington," *New York Times*, Monday, December 18, 2000. Alison Mitchell, "Results Official, at Last: Bush Defeats Gore," *New York Times*, Sunday, January 7, 2001.

[4]In general Buchanan would not be expected to attract much voter support according to standard models

of third-party candidates (Rosenstone, Behr, and Lazarus, 1984).

[5]Megan Garvey, "Bay Buchanan Sees Something Peculiar in Palm Beach Voting," *Los Angeles Times*, Friday, November 10, 2000. The story further observes that "longtime Reform members in the state described a party in 'disarray' with little organization, much less a groundswell of support for Buchanan in a place even he concedes is not his base."

[6]In 2000, Buchanan received 0.787% of the presidential vote in Palm Beach County while garnering only 0.3% of the overall Florida presidential vote. In contrast, Ross Perot, the Reform party candidate for president in 1996, received only 7.7% of the Palm Beach County vote while garnering 9.1% of the Florida vote. These data are from the Florida Department of State and are available at `http://election.dos.state.fl.us/online/elecresu.shtml`.

[7]In Brady et al. (2001) we list the early posters, including ourselves: Brady posted analysis on November 9, and Wand, Shotts, Sekhon, Mebane, and Herron posted on November 11. Lists of empirical work posted on the Internet through the end of November, 2000, appear at `http://www.bestbookmarks.com/election/` (created by Jonathan O'Keeffe), `http://economics.about.com/money/economics/library/weekly/aa111000.htm` (created by John S. Irons), `http://www.sbgo.com/election.htm` (created by Sebago Associates) and `http://madison.hss.cmu.edu` (created by Greg Adams and Chris Fastnow).

[8]The number 2,058 is the lower bound of a 99% confidence interval for the number of irregular Buchanan votes. "Who Won Florida? Are the Palm Beach Votes Irregular?" by Bruce E. Hansen, posted on November 11, 2000. Paper posted at `http://www.ssc.wisc.edu/~bhansen/vote/vote.html`.

[9]The final, certified results gave Bush 2,912,790 votes and Gore 2,912,253 votes in Florida. A few days after the election, the Associated Press reported a 327 vote margin based on the initial, automatic recount across Florida. Daniel J. Wakin, "Bush Campaign Pushes for a Gore Concession," *New York Times*, Friday, November 10, 2000.

[10]Reportedly LePore "split the names over two pages to make the type larger." Two days after the election LePore was quoted as saying, "Hindsight is 20-20, but I'll never do it again." Joel Engelhardt, "Elections Chief on Firing Line," *Palm Beach Post*, Thursday, November 9, 2000.

[11]Sinclair, Mark, Moore, Lavis, and Soldat (2000) report experimental evidence showing that a double-

column ballot format like the one used in Palm Beach County can be more confusing and cause more voter errors than a single-column ballot. Beyond that paper, published research on the effects of ballot design is not extensive and does not provide much guidance regarding the errors the Palm Beach County butterfly ballot may have induced voters to make (Campbell and Miller, 1957; Darcy, 1986; Hamilton and Ladd, 1996).

[12]The Court's ruling states, "even accepting appellants' allegations, we conclude as a matter of law that the Palm Beach County ballot does not constitute substantial noncompliance with the statutory requirements mandating the voiding of the election" (Supreme Court of Florida, Fladell, et al. Vs. Palm Beach County Canvassing Board, etc. et al. Case Nos. SC 00-2373 and SC 00-2376). The cases did not progress to hearings regarding the facts.

[13]Western (1995) provides a practically minded review of robust estimation. He suggests an approach to robust estimation of generalized linear models. He also discusses high-breakdown estimation of linear models using least median of squares (Rousseeuw, 1984). Christmann (1994) discusses application of LMS to a grouped binomial model (albeit without overdispersion).

[14]The threshold we use to declare an observation to be an outlier corresponds to the value of one of the tanh estimator's basic tuning parameters (c in Appendix equation (12)). If the magnitude of an observation's studentized residual is greater than the threshold, then the observation has zero weight in the tanh estimate of the overdispersed binomial regression model's coefficients. In effect, such an observation is omitted from the data used to estimate the parameters. Observations that have studentized residuals of magnitude less than c but greater than another tanh estimator tuning value, p, are downweighted.

[15]In the original version of this paper we found outlier counties that resulted from innocuous data reporting problems. The most notable example of this is Mississippi County, AR, which we found to have the most irregular Buchanan vote return in the country in our original work (posted on the web November 11, 2000, and reproduced in Figure 1 of Brady et al. (2001)). That finding was the result of a data reporting error propagated by both the CNN and ABC news websites from which (at that time) we were obtaining vote return data.

[16]We ignore undervotes (no apparent vote recorded on the ballot), overvotes (votes for more than one presidential candidate on a single ballot) and other spoiled ballots. For discussions of undervotes and overvotes in Palm Beach County see Joel Engelhardt and Scott McCabe, "Election 2000: Under-votes Could

Have Meant Victory for Gore," *Palm Beach Post*, Sunday, March 11, 2001, and Joel Engelhardt and Scott McCabe, "Election 2000: Over-votes Cost Gore the Election in FL," *Palm Beach Post*, Sunday, March 11, 2001.

[17]The 2000 Census data were built from Census 2000 Redistricting Data (Public Law 94-171) Summary File, Matrices PL1, PL2, PL3, and PL4 (U.S. Census Bureau. 2001. "FactFinder Tables." Accessed April 7, 2001, at `http://factfinder.census.gov`).

[18]We standardize the residuals of the demographic variables to have variance equal to 1.0 before computing the principal components.

[19]Our collection of counties (or county equivalents) includes the counties of all states except Michigan and Hawaii. In Michigan Buchanan appeared only as a write-in candidate. D.C.'s one county and Hawaii's four counties are too few for us to analyze. We pooled the data for Connecticut, Delaware and Rhode Island (which have respectively eight, three and five counties), using dummy variables to give each state a different intercept but requiring the other coefficients to be the same for all three states. For Alaska we use the 25 county-equivalents defined by the U.S. Census Bureau for reporting 1990 Census data.

[20]See the Appendix (Section 8) for data source information.

[21]Throughout the rest of this paper the word "residual" always refers to the studentized residual $\tilde{r}_i$ of Appendix equation (5), unless otherwise indicated.

[22]Whiskers extend to the nearest value not beyond 1.5 times the interquartile range. The residuals for the pooled states CT, DE and RI are omitted.

[23]In most states the distribution of the residuals that are not outliers is well described by a t-distribution with degrees of freedom equal to the number of counties minus the number of parameters.

[24]See the December 28, 2000 issue of *The Beaufort Gazette* for allegations regarding the Tillman precinct.

[25]"One Election Result Confirmed; Second Election Ruled a Do-over." *Associated Press State and Local Wire*. December 13, 2000.

[26]The county maps used to assess geographic contiguity appear in U.S. Bureau of the Census (1983).

[27]The existence of such clusters of outlier counties reinforces the importance of using a robust estimator that has a high breakdown point.

[28]"Voters Complain Around the Country," *Associated Press*, Thursday, November 9, 2000. B. Drummond Ayres, Jr., "St. Louis Sees Specter of Vote Fraud," *New York Times*, Sunday, March 4, 2001.

[29]"Judge Won't Rule on Reform Party." *Associated Press*. August 30, 2000.

[30]Pam Belluck, "Georgia Could Be Big Test for Bush," *New York Times*, Sunday, September 17, 2000.

[31]The race, Hispanic ethnicity and population variables are taken from the 1990 Census instead of the 2000 Census.

[32]According to 2000 Florida Statutes title IX chapter 101.62, a registered voter need not give any reason for requesting an absentee ballot.

[33]The data are based on certified numbers from the Florida Department of State and precinct-level returns provided by the 67 Florida counties. We used the precinct data to calculate the absentee returns.

[34]The election-day minus absentee difference in Palm Beach County is 0.00634. Liberty, Calhoun, Hamilton and Dixie counties have differences respectively of 0.0177, 0.00959, 0.00731 and 0.00706, based on total numbers of ballots cast for president respectively of 2,410, 5,174, 3,964 and 4,666. Union and Baker counties have the next largest differences after Palm Beach County. For both counties the proportion difference is 0.00620. The numbers of ballots cast in those counties are, respectively, 3,826 and 8,154.

[35]The containment within the inverse square root bounds is an implication of the binomial model we use more explicitly in the next paragraph, as long as the overdispersion measured by σ^2 is not too large.

[36]It is worth noting that even if Republicans and Democrats were equally likely to vote mistakenly for Buchanan, the net effect of the butterfly ballot would strongly favor Bush since Palm Beach County is heavily Democratic.

[37]See `http://www.joeldeckard.com/issues.htm` (Deckard), `http://www.ronhoward.org/mcissues.htm` (McGuire), `http://www.sherreelowe.com/` and `http://election.dos.state.fl.us/cgi-bin/CandHtml.exe?account=31167&elecid=20001107-GEN` (Lowe) (all accessed April 7, 2001).

[38]In the analysis of all precincts, reported in Table 2, 443 of the 515 precincts had $w_i = 1$ and 10 of the 515 precincts had $w_i = 0$, indicating that they were outliers. See Appendix equation (13) for the definition of w_i.

[39]In addition, Elms and Brady (2001) show that the extra Buchanan vote is spread all over Palm Beach County and there is a precipitous drop in the proportion of the Buchanan vote in precincts in adjoining counties.

[40]The ballot data were acquired from the Palm Beach County Supervisors of Elections.

[41]In the absence of grouping there cannot be overdispersion (McCullagh and Nelder, 1989, 125) and high-breakdown estimators do not exist. To summarize, Christmann (1994) shows that for non-grouped binary data no estimator exists that has a high breakdown point and produces the estimate $\pi_i = 1$ if $y_i = 1$ for all i and the estimate $\pi_i = 0$ if $y_i = 0$ for all i. M-estimators, including maximum likelihood, have the latter property.

[42]This number is calculated by $269835 \times 0.0085 = 2294$. The 269,835 number is the total number of votes that Nelson (D) received in the entire county on election day, including the precincts missing from the ballot image data.

[43]Megan Garvey, "Bay Buchanan Sees Something Peculiar in Palm Beach Voting," *Los Angeles Times*, Friday, November 10, 2000.

[44]Another allegation related to the butterfly ballot is that the Palm Beach County ballot led to excessive overvoting in the presidential race. The subject of overvoting is beyond the scope of this paper (recall footnote 16; also see Tyler Bridges, "Ballots Offer Clues on Intent: Experts See Patterns in the Overvotes," *Miami Herald*, Friday, May 11, 2001).

[45]"About This Project," *Miami Herald*, Friday, May 11, 2001.

[46]Martin Merzer, " 'Overvotes' Leaned to Gore, But to Win, He Needed Help of Dimpled Ballots," *Miami Herald*, Friday, May 11, 2001. Also see Martin Merzer, "Review Shows Ballots Say Bush, But Gore Backers Have Some Points to Argue," *Miami Herald*, Wednesday, April 4, 2001.

[47]Intuitively, the finite sample breakdown point of an estimator is the smallest proportion of the observations that must be replaced by arbitrary values in order to force the estimator to produce values arbitrarily far from the parameter values that generated the original data (Donoho and Huber, 1983).

[48]Strictly speaking, direct comparability holds asymptotically in the number of counties in each state, subject to various assumptions. Differences due to the variation of degrees of freedom across states, as in the t-distributions we noted in footnote 23 are negligible compared to the variation caused by the seriously anomalous processes that occurred in some counties.

[49]To be technically precise, the statistical theory that supports the development of tanh and LQD estimators assumes the existence of a *Fisher-consistent* estimator (Hampel et al., 1986, 83).

[50]In the overdispersed binomial model there is information about the constant from $[\hat{\pi}_i(1-\hat{\pi}_i)]^{-1/2}$ in r_i^*.

[51]See `http://jsekhon.fas.harvard.edu/rgenoud/` for an **R** version of the GENOUD software and

http://jsekhon.fas.harvard.edu/genoud/ for more information.

[52]For details see Hampel et al. (1981, 645) or Hampel et al. (1986, 160–165).

[53]Hampel et al. (1981) use k for the tuning parameter of ψ that we have denoted by d. The same information about the tuning parameters appears in (Hampel et al., 1986, 163, Table 2) with notation r and k used for the parameters we have denoted by c and d.

[54]Full iteration makes the weighted estimator equivalent to the M-estimator (Hampel et al., 1986, 116).

[55]http://www.vote.nyc.ny.us/BoePages/Results/2000General/allg2000.pdf (accessed February 21, 2001).

[56]http://vote2000.ss.ca.gov/Returns/pres/59.htm (accessed January 21, 2001).

[57]http://www.state.de.us/election/index.htm (accessed June 26, 2001).

[58]http://enight.dos.state.fl.us/report.asp?Date=001107 (accessed January 21, 2001).

[59]http://www.sboe.state.nc.us/y2000elect/stateresults.htm (accessed January 22, 2001).

[60]http://www.state.ok.us/~elections/00result.html (accessed April 4, 2001).

[61]http://vermont-elections.org/2000geresults.htm (accessed January 22, 2001).

[62]http://soswy.state.wy.us/election/2000/results/g-usp.htm (accessed January 22, 2001).

[63]http://www.gov.state.ak.us/ltgov/elections/elect00/00genr/index.shtml, (accessed June 19, 2001).

[64]http://www.gov.state.ak.us/ltgov/elections/results/sov.zip, (accessed June 19, 2001).

[65]http://factfinder.census.gov/servlet/DTGeoSearchByListServlet?ds_name=DEC_2000_PL_U&state=dt&_lang=en&_ts=11623041369 (accessed June 19, 2001).

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Table 1: Contiguous Counties Among Those with the Largest Studentized Residuals

State	County Name
OH	Hocking
OH	Athens
MD	Wicomico
MD	Somerset
SD	Union
IA	Sioux
NE	Dixon
NE	Thurston
NE	Dakota
IA	Plymouth
IA	Woodbury
MT	Deer Lodge
MT	Silver Bow
KS	Wabaunsee
KS	Shawnee
KS	Pottawatomie
CO	Arapahoe
CO	Jefferson
CO	Adams
KY	Kenton
KY	Boone
WI	Wood
WI	Lincoln
WI	Marathon
KS	Wyandotte
MO	Jackson
OH	Belmont
WV	Ohio
WV	Marshall
OH	Jefferson
WV	Brooke
WV	Hancock
MO	St. Louis City
MO	St. Louis
MN	Ramsey
MN	Hennepin

Notes: Results based on 3,049 counties. This table presents all contiguous counties with studentized residuals of magnitude greater than or equal to 4.0.

Table 2: Votes for Reform Candidates by Proportion Voting for U.S. Senate Candidates, for Palm Beach County Precincts

Scope	Candidate	Intercept	Nelson (D)	Deckard (Ref)	$\hat{\sigma}$
All Precincts	Buchanan	−6.17 (0.15)	2.06 (0.21)	−12.74 (14.79)	1.22
District 35	Buchanan	−7.48 (0.51)	3.85 (0.71)	13.13 (23.89)	1.26
District 35	Lowe	−1.98 (0.34)	−1.86 (0.51)	18.06 (14.46)	1.54
District 16	Buchanan	−7.00 (0.29)	3.32 (0.46)	3.95 (15.09)	1.15
District 16	McGuire	−3.37 (0.30)	−1.00 (0.50)	25.96 (9.10)	1.52

Note: Entries are tanh estimates of coefficient parameters of the overdispersed binomial regression model using precinct-level data from the 2000 election (standard errors in parentheses). The last column reports the LQD dispersion estimate $\hat{\sigma}$. Number of precincts for each scope: all precincts, 515; District 35, 105; District 16, 149.

Table 3: Vote for Buchanan by U.S. Senate Vote in Palm Beach County, for Individual Ballots by Ballot Type

Ballot Type	Intercept	Nelson (D)	Deckard (Ref)
Election-Day	−5.18 (0.034)	0.61 (0.040)	2.41 (0.138)
Absentee	−6.11 (0.156)	−0.21 (0.236)	3.68 (0.400)

Note: Entries are maximum likelihood estimates of coefficient parameters of the binary logistic regression model using ballot data from the 2000 election (standard errors in parentheses). Ballots with spoiled presidential votes (under-votes or over-votes) are omitted. Including them does not materially change the results. Number of unspoiled ballots for each type: election-day, 381,449; absentee, 36,412.

Table 4: Proportion Voting for Buchanan by U.S. Senate Vote Choice and Ballot Type in Palm Beach County

	Election-Day Ballots		Absentee Ballots	
Senate Candidate	Proportion	*N*	Proportion	*N*
Bill Nelson (D)	0.0102	228,455	0.0017	17,779
Joel Deckard (Ref)	0.0590	1,000	0.0808	99

Note: Entries are the proportion of ballots with a vote for Buchanan out of the *N* ballots of each type voted for each Senate candidate, using ballot data from the 2000 election. Ballots with spoiled presidential votes (under-votes or over-votes) are omitted.

Table 5: Counties with the Largest Positive and Largest Negative Studentized Residuals, 2000 National Analysis

State	County Name	Studentized Residual	Vote Proportion Expected	Vote Proportion Actual	Number of Ballots	Order
FL	Palm Beach	36.14	0.0010	0.0079	433,186	1
SC	Jasper	28.26	0.0013	0.0379	6,469	2
KS	Pottawatomie	19.53	0.0096	0.0528	7,731	3
WV	Hancock	18.16	0.0048	0.0289	13,472	4
NV	Clark	15.88	0.0037	0.0080	381,845	5
WV	Brooke	15.57	0.0046	0.0286	9,405	6
OH	Jefferson	14.56	0.0066	0.0303	34,636	7
OH	Athens	14.23	0.0049	0.0292	25,447	8
MN	Hennepin	13.67	0.0010	0.0040	573,846	9
NE	Douglas	12.52	0.0018	0.0038	183,156	10
WI	Milwaukee	12.25	0.0010	0.0037	433,537	11
MN	Ramsey	11.67	0.0009	0.0057	244,278	12
IA	Woodbury	11.49	0.0038	0.0107	37,896	13
WV	Marshall	10.35	0.0054	0.0196	13,498	14
WV	Ohio	10.25	0.0031	0.0126	17,964	15
AZ	Maricopa	9.99	0.0057	0.0080	899,808	16
MT	Silver Bow	9.89	0.0100	0.0265	16,703	17
KS	Shawnee	9.67	0.0048	0.0097	74,373	18
MO	St. Louis	9.66	0.0012	0.0024	486,884	19
KS	Sedgwick	9.41	0.0034	0.0062	163,417	20
MO	St. Louis City	9.30	0.0005	0.0025	124,752	21
AR	Union	8.95	0.0052	0.0186	15,609	22
MO	Jackson	8.55	0.0014	0.0030	272,062	23
AR	Faulkner	8.50	0.0052	0.0148	29,216	24
IA	Plymouth	8.20	0.0043	0.0143	10,118	25
OH	Belmont	7.79	0.0074	0.0218	30,141	26
CO	Adams	7.60	0.0049	0.0093	107,852	27
CO	Jefferson	7.48	0.0034	0.0058	235,491	28
FL	Pinellas	7.47	0.0010	0.0025	398,472	29
NE	Dakota	7.33	0.0046	0.0144	6,061	30
WI	Marathon	7.22	0.0041	0.0116	58,374	31
NM	Bernalillo	7.19	0.0011	0.0019	204,319	32
SD	Minnehaha	7.13	0.0034	0.0062	61,369	33
NY	Erie	6.95	0.0050	0.0079	424,654	34
NE	Thurston	6.95	0.0063	0.0245	2,082	35
MD	Somerset	6.59	0.0025	0.0064	7,604	36
KS	Wyandotte	6.42	0.0025	0.0057	48,272	37
CO	El Paso	6.39	0.0047	0.0073	200,757	38
VA	Warren	6.35	0.0026	0.0088	11,166	39
NE	Dixon	6.31	0.0080	0.0233	2,792	40
WI	Lincoln	5.99	0.0046	0.0178	14,239	41

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Table 5: *continued from previous page*

State	County Name	Studentized Residual	Vote Proportion Expected	Vote Proportion Actual	Number of Ballots	Order
TX	Harris	5.84	0.0014	0.0017	974,822	42
OH	Ashland	5.81	0.0061	0.0176	21,258	43
IA	Dubuque	5.79	0.0040	0.0074	40,323	44
PA	Allegheny	5.46	0.0024	0.0036	582,478	45
KY	Boone	5.41	0.0021	0.0043	31,984	46
GA	Whitfield	5.35	0.0045	0.0081	23,169	47
WI	Wood	5.34	0.0048	0.0123	35,761	48
KY	Kenton	5.10	0.0021	0.0036	55,987	49
MS	Leflore	5.09	0.0012	0.0039	11,276	50
CO	Arapahoe	4.96	0.0031	0.0048	189,942	51
MS	Washington	4.85	0.0012	0.0033	18,328	52
IA	Sioux	4.84	0.0018	0.0056	14,692	53
KS	Wabaunsee	4.76	0.0098	0.0260	3,420	54
AK	Matanuska-Susitna	4.60	0.0191	0.0311	21,254	55
MT	Deer Lodge	4.55	0.0101	0.0254	4,534	56
MS	Wilkinson	4.54	0.0022	0.0078	4,098	57
IN	Allen	4.48	0.0059	0.0083	114,320	58
KY	Graves	4.25	0.0032	0.0063	14,212	59
WI	Brown	4.22	0.0032	0.0060	107,769	60
SC	Abbeville	4.19	0.0022	0.0084	8,374	61
SD	Union	4.19	0.0100	0.0189	5,772	62
MD	Wicomico	4.09	0.0023	0.0042	31,795	63
AL	Lauderdale	4.07	0.0056	0.0092	32,137	64
GA	Richmond	4.06	0.0030	0.0044	57,359	65
OH	Hocking	4.06	0.0078	0.0202	10,756	66
IA	Benton	4.02	0.0038	0.0081	11,766	67
TX	Cottle	4.01	0.0023	0.0132	757	68
...						
GA	Fulton	−4.08	0.0028	0.0021	261,945	3042
WY	Albany	−4.12	0.0131	0.0054	13,163	3043
WY	Teton	−4.35	0.0132	0.0033	9,667	3044
LA	East Baton Rouge	−4.37	0.0072	0.0041	168,989	3045
OR	Deschutes	−4.37	0.0074	0.0042	57,885	3046
AL	Madison	−5.61	0.0056	0.0028	113,318	3047
LA	Caddo	−6.21	0.0105	0.0035	95,639	3048
LA	Orleans	−8.89	0.0112	0.0035	181,221	3049

Notes: Results based on 3,049 counties. This table presents all counties with studentized residuals greater than or equal to 4.0 or less than or equal to −4.0.

Figure 1: The Palm Beach County Butterfly Ballot

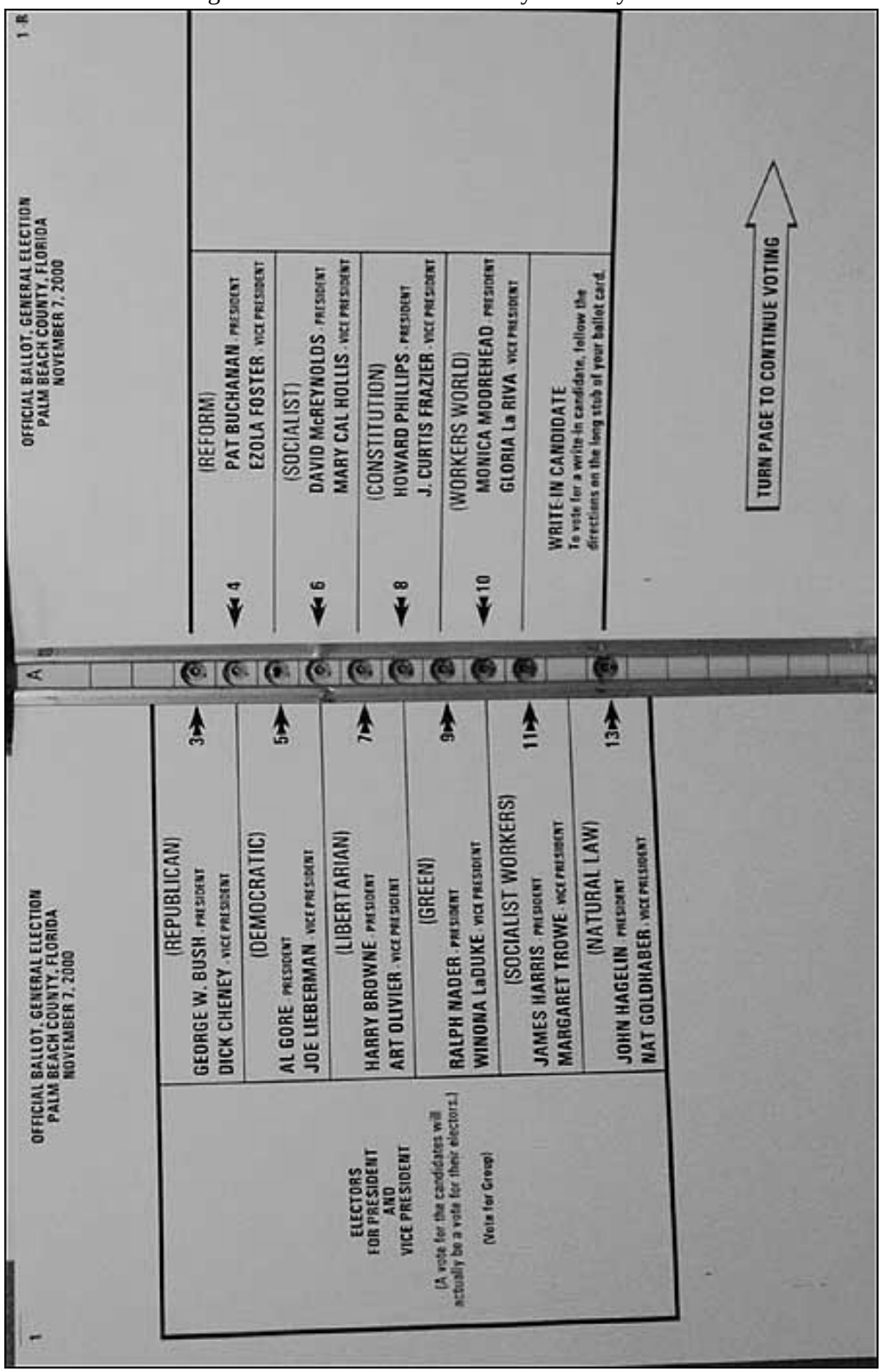

Figure 2: Boxplots of Studentized Residuals in United States Counties, by State

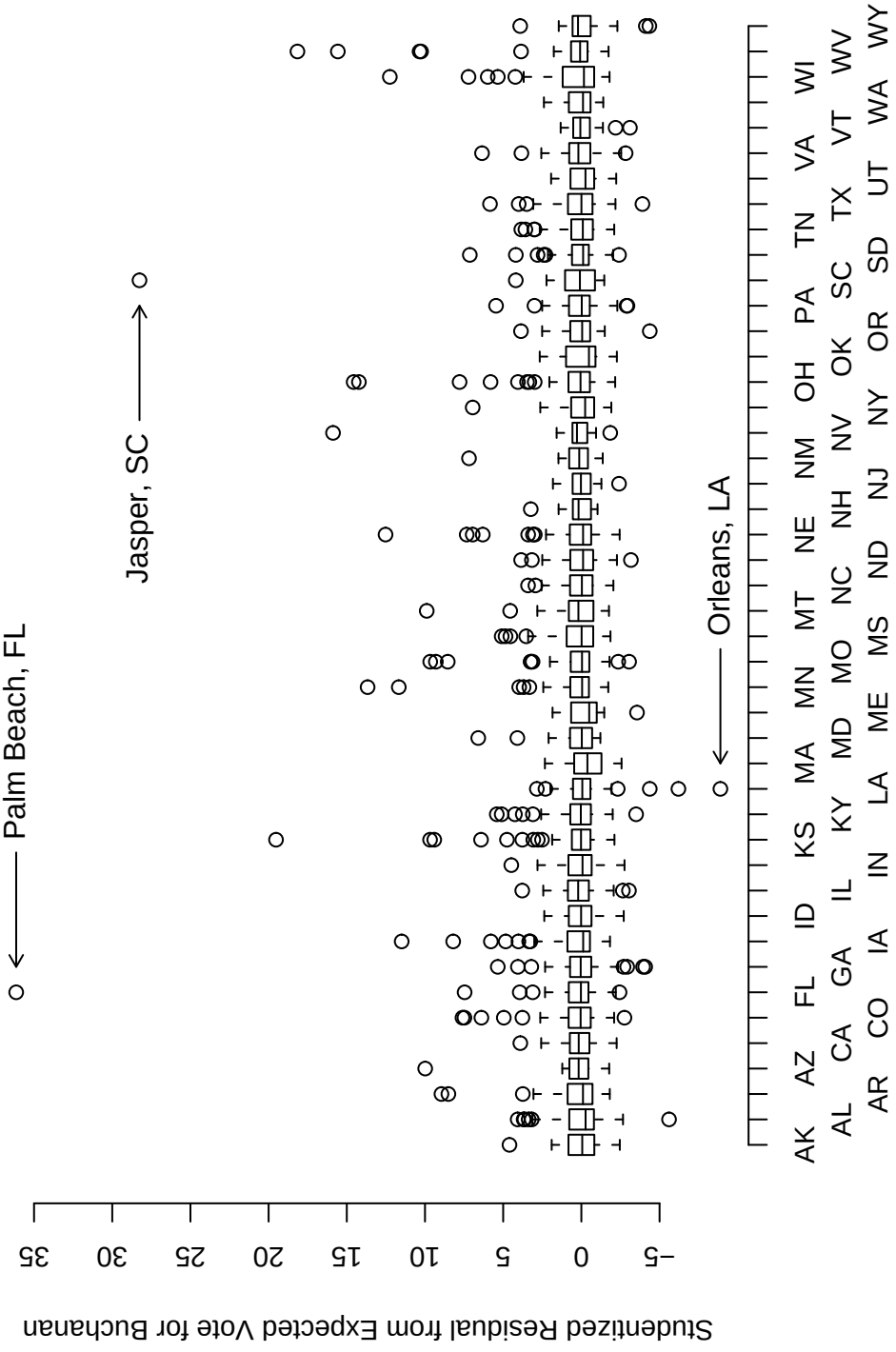

Figure 3: 2000 Election-Day Minus Absentee Support for Buchanan in Florida Counties